Strongly coupled Bose-Fermi mixtures in 2d

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Motivation

Why study mixtures !?

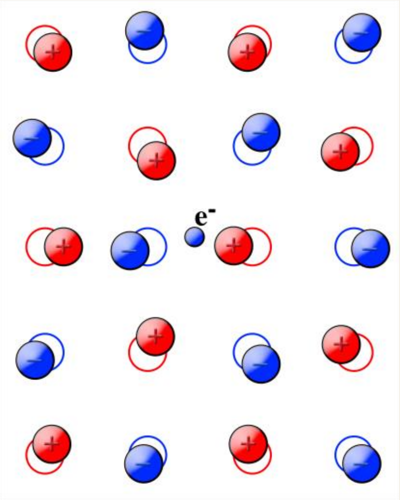
- Standard paradigm of quantum matter: weakly interacting quasiparticles (= perturbation theory).
  - E.g.: Fermi Liquids, BCS superconductors…
- Many interesting cases beyond: topology, strong correlations…
- Encountered in solid state systems or ultracold atoms.
- Polaron / mixtures : examples of such systems!
  - Interesting in themselves and for engineering strong correlations.

Introduction: polaron

- Landau 1933:
  polaron = electron + lattice phonons.
  - E.g.: effective mass, mobility.

- In solids: weak coupling.

- Fröhlich model: linear coupling.



($\hat{c}$: electron, $\hat{a}$: phonon) $$\hat{V}_{e-ph} \sim \hat{c}_k^\dagger \hat{c}_{k-q} \hat{a}_q + h.c.$$

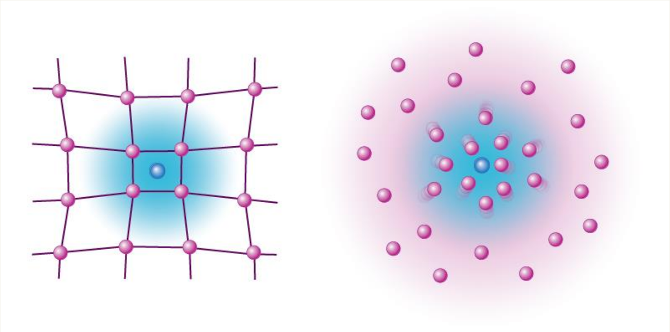
Extensively studied: weak/strong coupling expansion, Feynman path integral, Monte Carlo…

Devreese & Alexandrov R. Prog. Phys. '09, Devreese 1012.4576

“Modern” interest

- Laboratory for strongly correlated phases of matter:
  - Ultracold atoms.
  - 2d semiconductors and transition metal dichalcogenides (TMD).

Lattice polaron



Cold atom polaron

- Polaron = impurity in bath + cloud of excitations.

- Interplay of few-body and many-body physics: starting point for the study of mixtures.

Fermi polaron

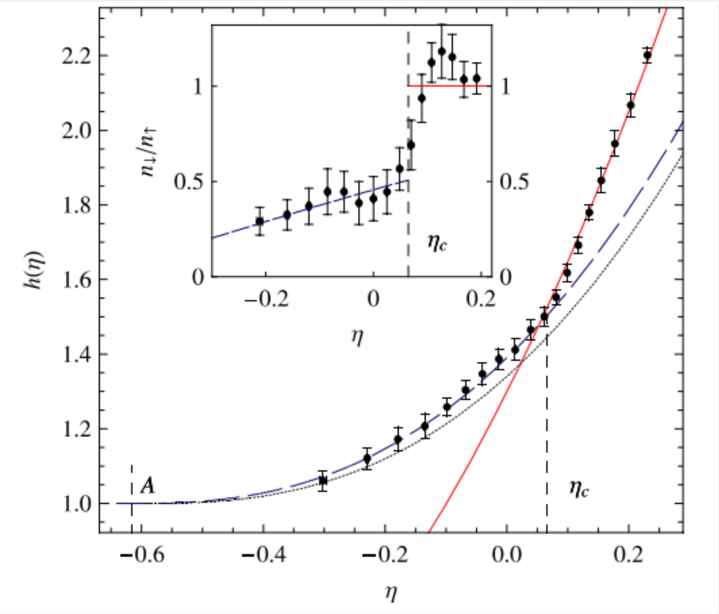
- Spin-imbalanced Fermi mixtures, $n_{\uparrow/\downarrow}$.
  - Motivation: interplay with BCS.
- Phases:
  - weak imbalance → superfluidity.
  - Strong imbalance → normal phase.

Limit $n_\uparrow \to 0$: impurity, polaron.

Phase diagram of normal phase:
Fermi Seas of majority + minority polarons!

[Chevy PRA ’06; Chevy & Mora, R. Prog. Phys. ‘10]

Equation of state at unitarity

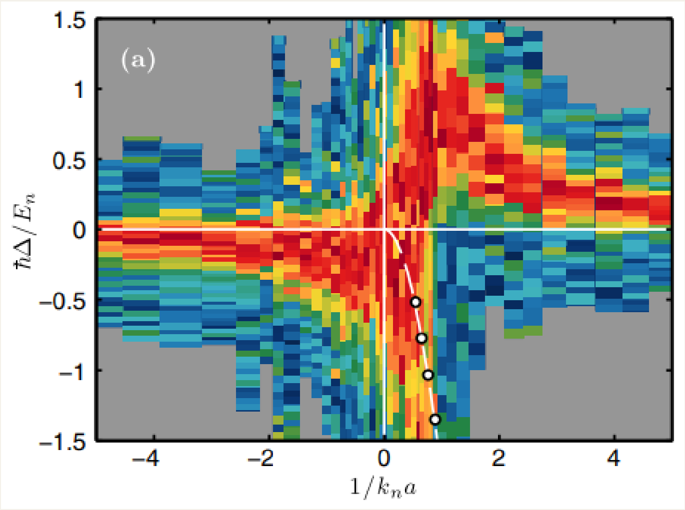


h: pressure, vs. $\eta: \mu_\uparrow/\mu_\downarrow$

Dots: experiment, dashed: polaron
Dotted: MC, red: superfluid
[Nascimbene et al. Nature 2010]

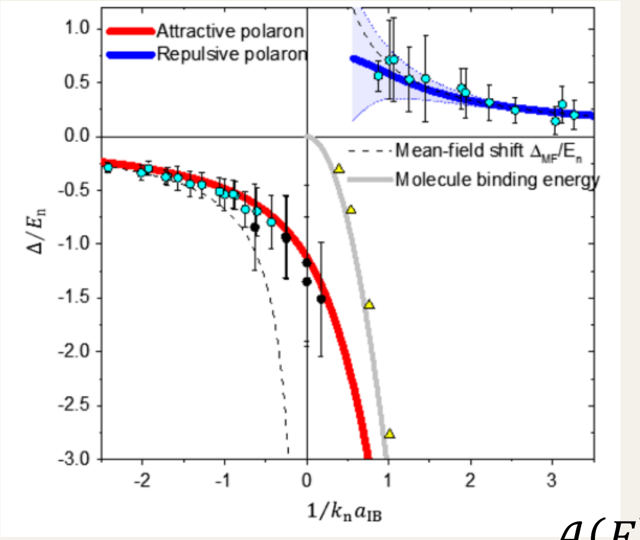
Bose polarons

- Fermi polaron studied on its own [Schirotzek, ..., Zwierlein PRL ‘09]
- Bose polaron: rich physics (Efimov, etc.)... but 3-body loss
- Measurements of spectral functions of impurities in BECs:



^{39}K in ^{39}K

[Jørgensen et al. PRL ’16]



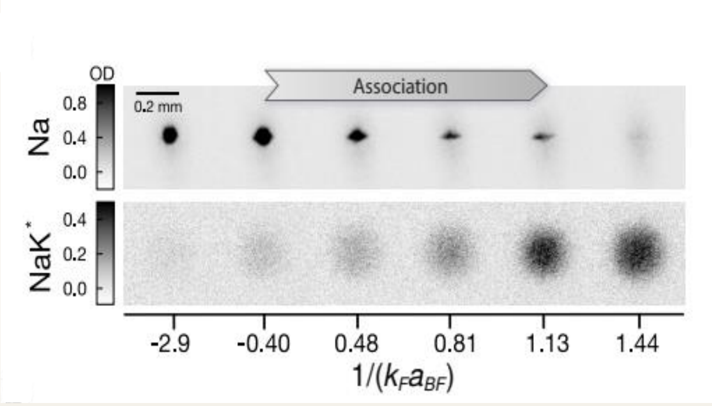
^{39}K in ^{87}Rb

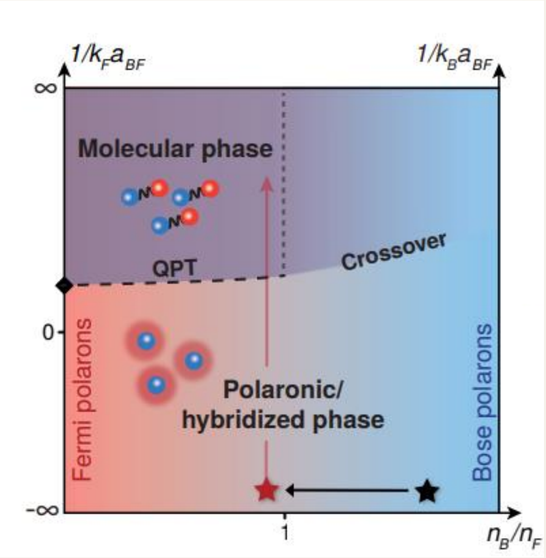
[Hu et al. PRL ’16]

$\mathcal{A}(E)$ vs a_{IB}^{-1}

…Bose-Fermi mixtures ?

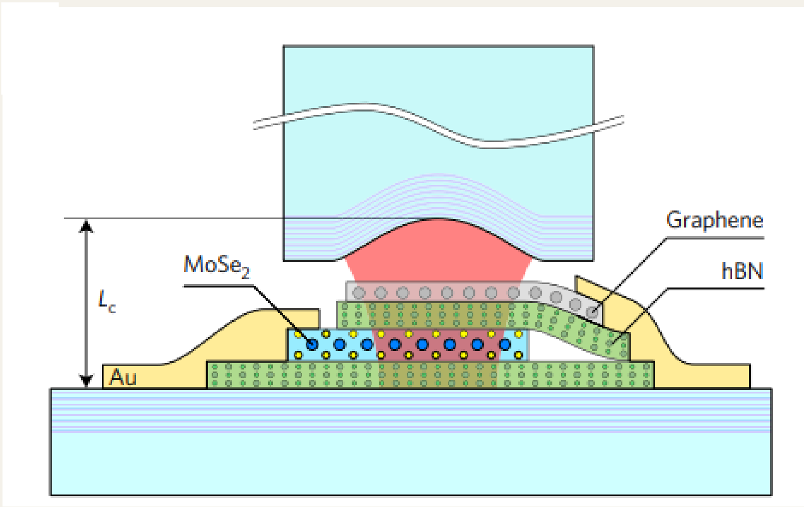
- Next step: mixtures!
- Realization of strongly-correlated phases.
- Phase transition between polaronic Na condensate and fermionic KNa molecules





[Duda, …Bloch et al., Nat. Phys. '23]

...and solid state



[Sidler, ..., Imamoglu Nat. Phys. ’16]

- 2*d* transition metal dichalcogenides / semiconductors.
- Bosons = excitons, composite of electron + hole.
- Fermions = free carriers.
- Observations : polaron, exciton fluid, checkerboard phases...

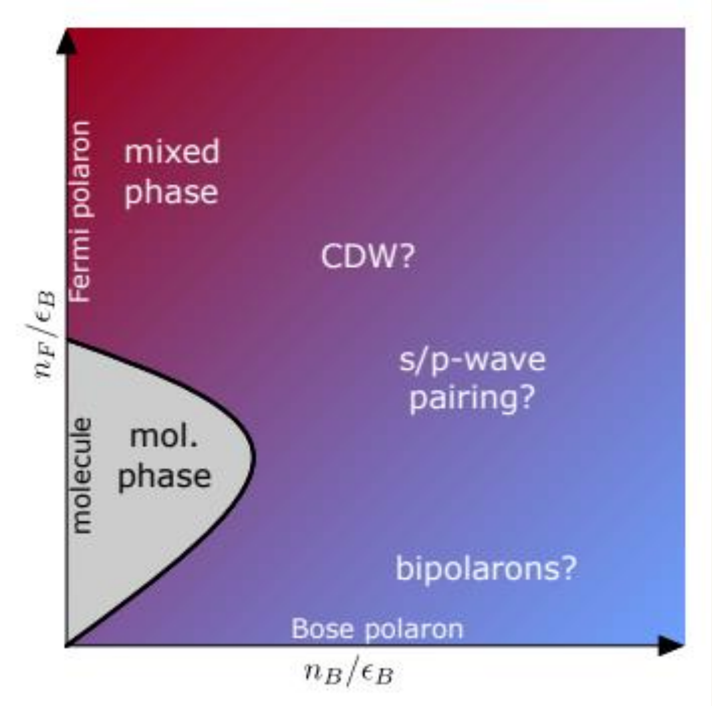
[Ma et al., Nature ‘21]
[Lagoin et al., Nat Mater. ‘23]

Rich phase diagram expected

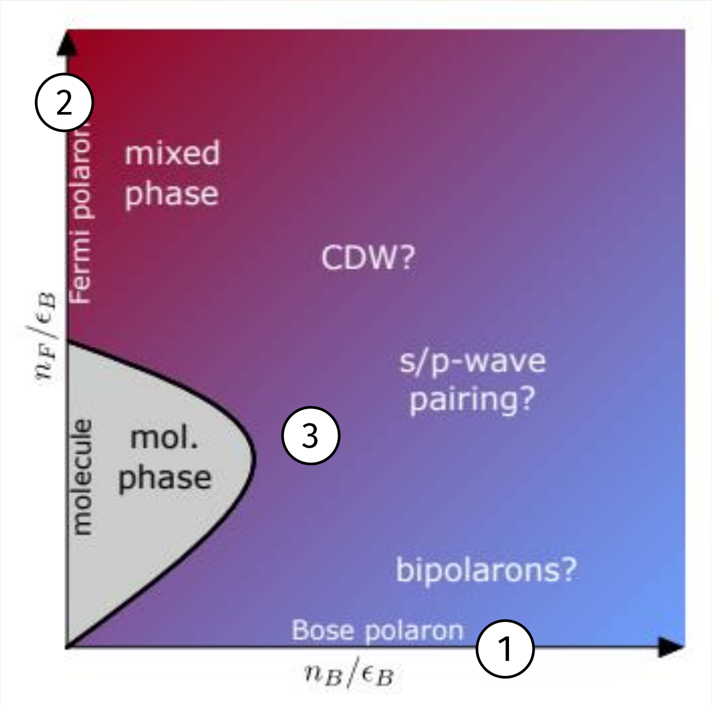
What do we know about Bose-Fermi mixtures?

- Phase diagram : studied in *3d.*
  - *p*-wave superconductivity.
    [Kinnunen et al. PRL ‘18]
  - Enhanced superfluidity.
    [Enss & Zwerger EPJ B ‘09]

- 2*d*? Enhanced quantum fluctuations.



2d phase diagram ?

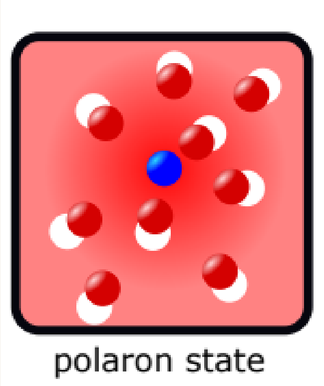


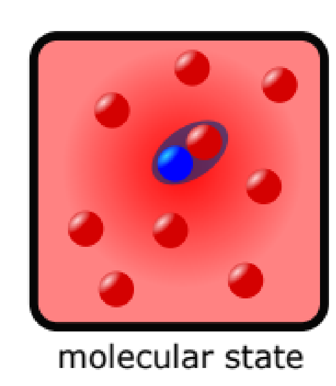
- Known limits:
- Bose polaron ①
  - Variational, diagrams
- Fermi polaron ②
  - Variational, renormalization, diagramatics
- Unexplored! ③

We focus on the Fermi polaron limit
I.e. extending from 2 to 3.

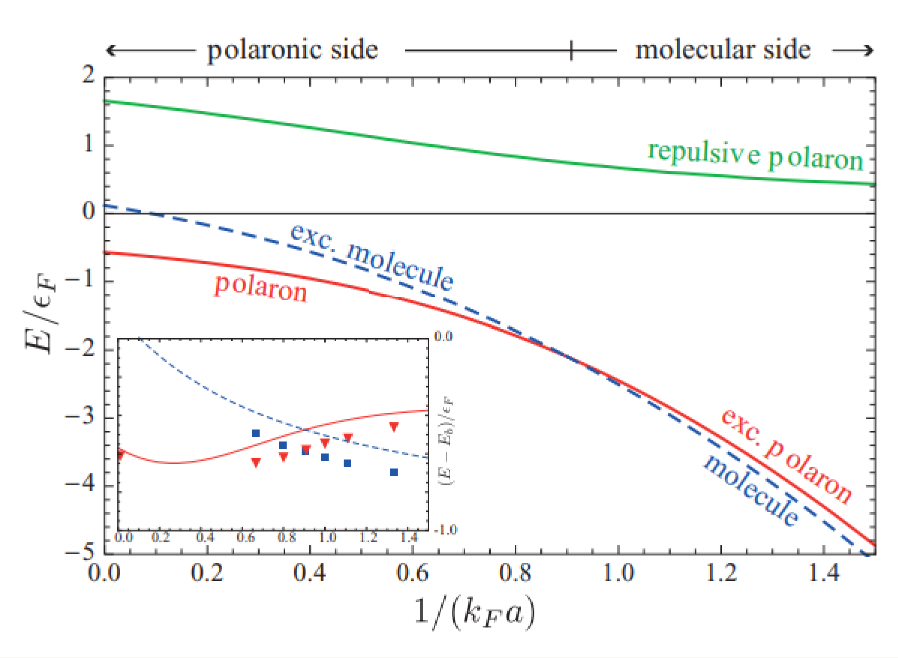
Start: Fermi polaron

- Ground states:
- Polaron $\sim \sum_{\boldsymbol{k},\boldsymbol{q}} b^\dagger_{\boldsymbol{p}+\boldsymbol{q}-\boldsymbol{k}} c^\dagger_{\boldsymbol{k}} c_{\boldsymbol{q}} |FS_N\rangle$
- Molecule $\sim \sum_{\boldsymbol{k}} b^\dagger_{\boldsymbol{p}-\boldsymbol{k}} c^\dagger_{\boldsymbol{k}} |FS_{N-1}\rangle$





3*d*: well understood!



[Schmidt & Enss, PRA '11]

Importance of bound state

- $d \leq 2$: Any attractive interaction → bound state, ϵ_B!
  - Theory needs to take it into account.

Importance of bound state

- $d = 2$ Fermi polaron:
  Basic variational → no transition!

  [Zöllner et al. PRA ‘11]

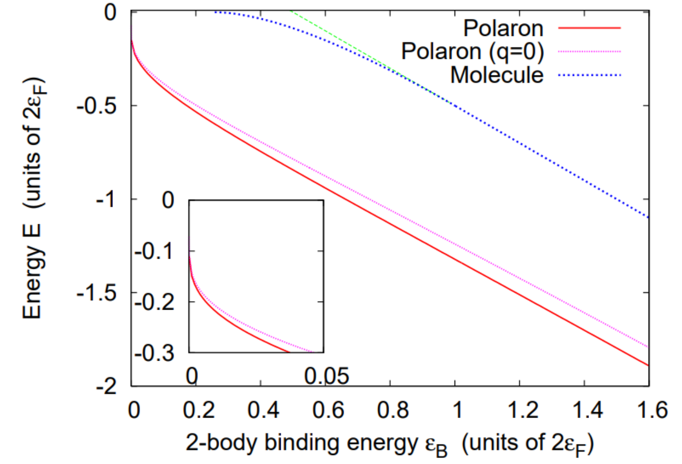
- Contradiction with Monte Carlo.

  [Bertaina AIP ‘12 ; Kroiss & Pollet PRB ‘14; Vlietnick et al. PRA ’14]

- 3-body terms in molecule necessary!

  [Parish PRA ‘11]

E(polaron/molecule) vs. ϵ_B

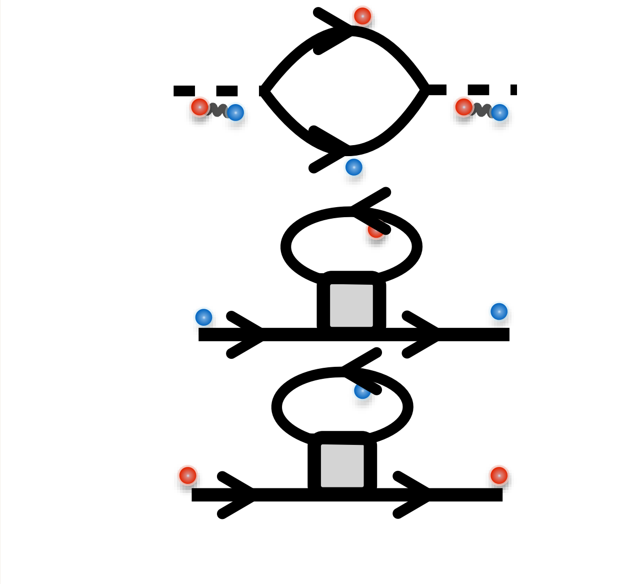


Consequences for field theory

- To describe strongly-correlated mixture: field theory
- Bound state: also an issue!
- T-matrix approach:
  works in 3d, not in 2d!
- Equivalent to Chevy variational method.

[Combescot et al. PRL ‘07]

[Schmidt, Enss, Pietilä, Demler PRA 12]



Field theoretical approach

- Action:

$$S = \int_x \psi^* \left(\partial_\tau - \frac{\nabla^2}{2m_F} - \mu_\psi \right) \psi + \phi^* \left(\partial_\tau - \frac{\nabla^2}{2m_B} - \mu_\phi \right) \phi + g\psi^*\phi^*\phi\psi$$

Interaction

ϕ:boson, ψ: fermion

Quartic interaction: extended Fröhlich model

Fröhlich: expansion about condensate

Two channel action

ϕ:boson, ψ: fermion

t: molecule field!

$$S = \int \psi^* G_\psi^{-1} \psi + \phi^* G_\phi^{-1} \phi + t^* G_t^{-1} t + h(\psi^* \phi^* t + \text{h.c.})$$

G: propagators

Fermion + boson bind into molecule

- Equivalent when $h \to \infty$.
- Easier to treat molecule: lower-order correlations.

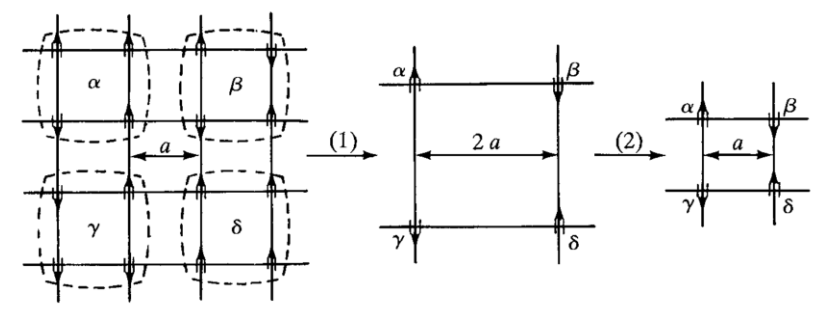
Renormalization group concept

- Strongly correlated problem:
  many degrees of freedom correlated over all scales.

- Renormalization group: integrate iteratively dofs from short to long distances.

  E.g. spin block RG

  → Running Hamiltonian.



- The running Hamiltonian contains all couplings allowed by symmetry.
- Renormalization group: integrate iteratively dofs from short to long distances.

Effective action formalism

FRG: implementation of Wilsonian RG
- in momentum space
- At the level of the effective action

$$\mathcal{Z}[J] = \int \mathcal{D}[\varphi] \exp\left(-S[\varphi] + \int_x J\varphi\right), \qquad \Gamma[\phi] = -\ln \mathcal{Z}[J] + \int_x J\phi.$$

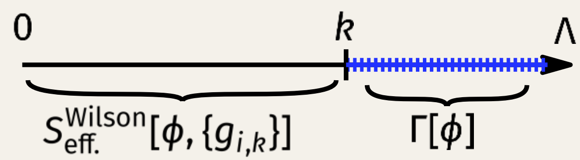
Example with ϕ^4 theory; J: external source.

- Vertices $\delta^n\Gamma/\delta\phi^n$: physical information
  - $\Gamma(\phi \to \text{const.}) = U(\phi)$: effective potential → thermodynamics
  - $\Gamma^{(2)} = [G]^{-1}$: inverse propagator.

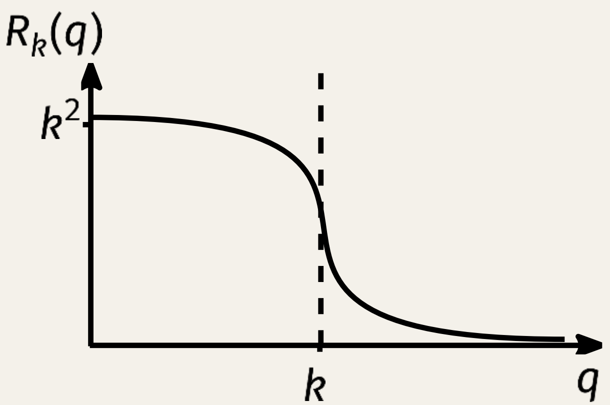
(NB: fermions = Grassman variables: Γ only meaningful through expansion)

The FRG in a nutshell

- Similar in concept to Wilsonian RG, degrees of freedom are progressively integrated out.



- Implemented by adding to S a “mass-like” term:



$$S \to S_k = S + \Delta S_k,$$

$$\Delta S_k[\varphi] = \frac{1}{2}\int_q \varphi(q) R_k(q) \varphi(q).$$

R_k: modes at momenta $\lesssim k$ get a very large mass.

- k-dependent effective action $\Gamma \to \Gamma_k$.

$$\Gamma_{k=\Lambda} = S \xrightarrow{\text{RG flow}} \Gamma_{k=\Lambda} = \Gamma$$

Exact flow equation:

$$\partial_k \Gamma_k[\phi] = \frac{1}{2}\,\mathrm{Tr}\left\{\partial_k R_k \left(\Gamma_k^{(2)}[\phi] + R_k\right)^{-1}\right\}$$

[Wetterich, PLB ’93; Morris, IJMP ’94; Ellwanger, ZPC ’94]

Application to mixtures

Ansatz: Gradient + vertex expansion

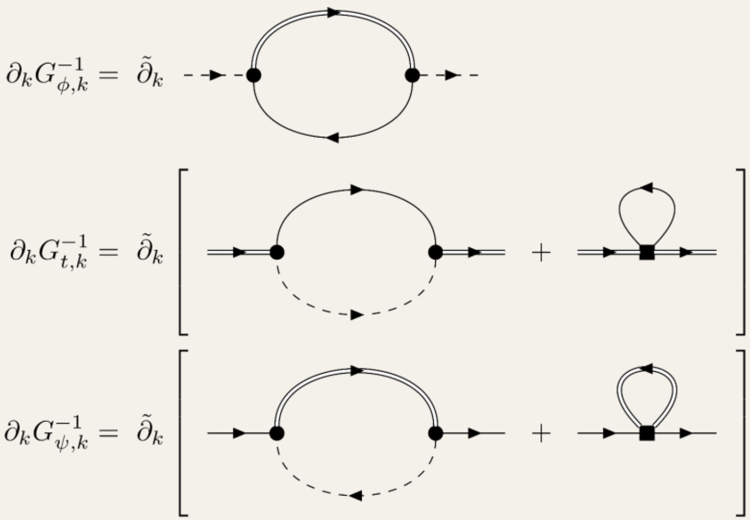
$$\Gamma_k = \int_P \left\{ \psi_P^* G_{\psi,k}^{-1}(P)\psi_P + \phi_P^* G_{\phi,k}^{-1}(P)\phi_P + t_P^* G_{t,k}^{-1}(P) t_P \right\} + \int_X h_k(\psi_X^*\phi_X^* t_X + \text{h.c}) + \lambda_k \psi_X^* t_X^* t_X \psi_X$$

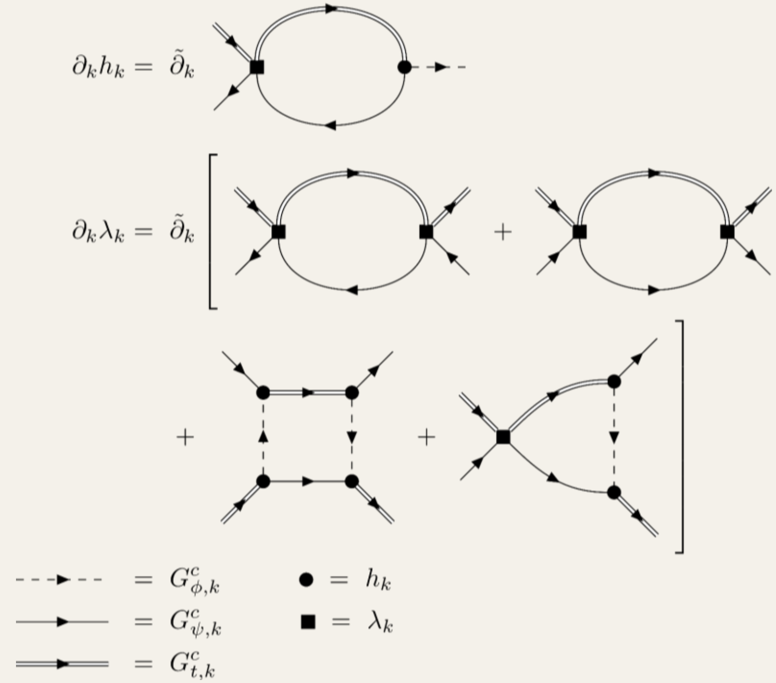
- Incorporates 2-body effects
- Running propagators, include:
  - quasiparticle weight;
  - detuning.

Fermion-molecule interaction: 3-body effect!

Flow equations

Projection of the flow of Γ
~all possible 1-loop diagrams



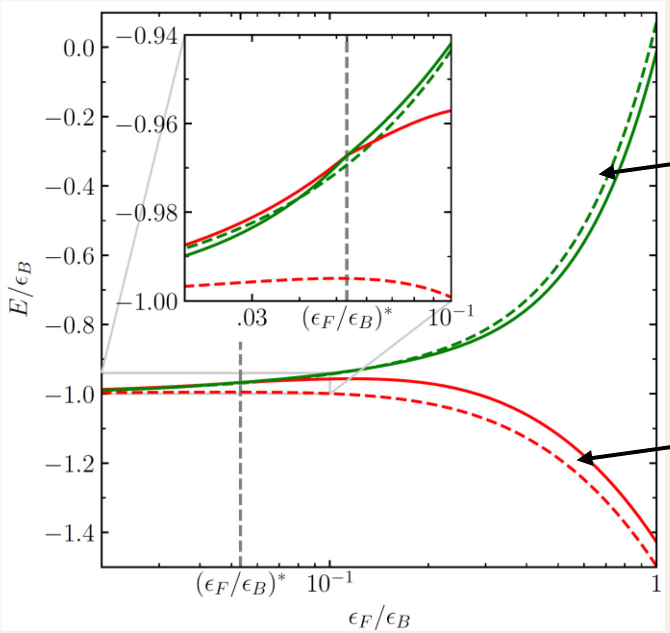


2*D* Polaron-molecule transition

polaron / molecule energies vs ϵ_F/ϵ_B

Green : molecule energies
Red : polaron energies

Dashed / solid :
without/with three-body correlations



3-body correlations crucial to recover transition!

Results: transition position

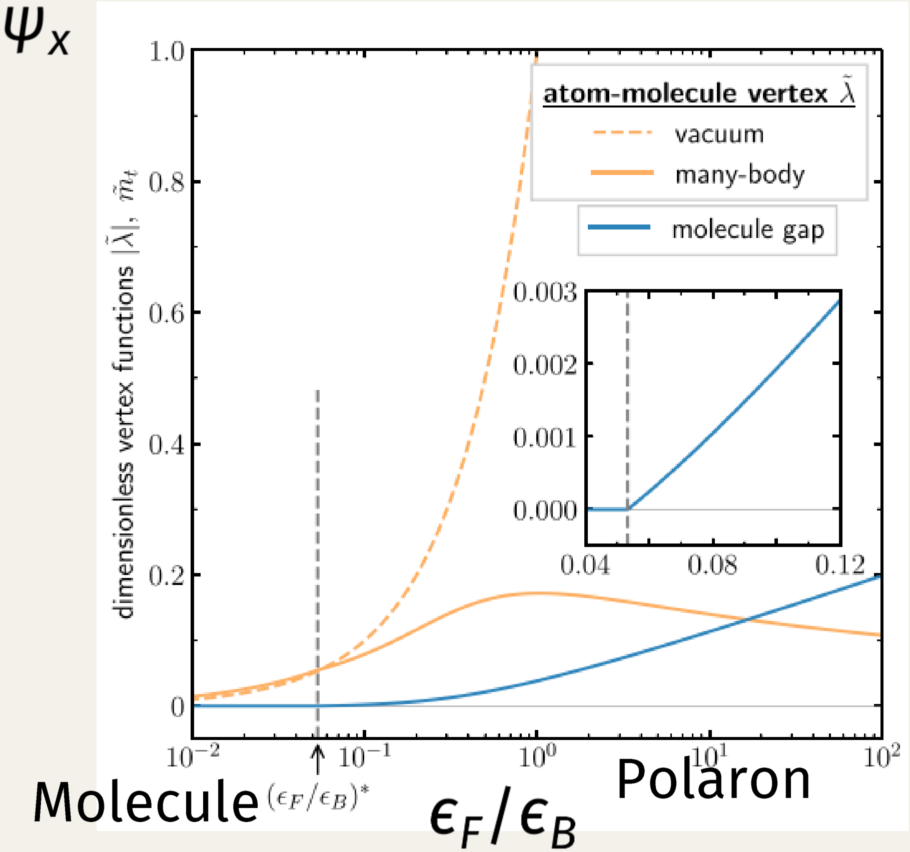
	Theoretical approach	$(\epsilon_B/\epsilon_F)^*$	$-\log(\epsilon_B/2\epsilon_F)/2 = \log(k_F a_{2D})$
	fRG (present work)	18.78	-1.12
[Parish PRA '11]	Basic variational [56]	9.9	-0.8
[Parish Levinsen PRA '13]	High-order variational [59]	14	-0.97
[Kroiss Polett PRB '14]	Diag. MC [60]	18.1 ± 7.2	-1.1 ± 0.2
[Vlietink et al PRB '14]	Diag. MC [61]	13.4 ± 4	-0.95 ± 0.15
[Bertaina AIP '12]	Diffusion MC [58]	≈ 15	≈ -1
[Koschorreck et al Nature '12]	Experiment [113,114]	11.6 ± 4.6	-0.88 ± 0.2

Good agreement with other theoretical predictions + experiment!

Results for the vertices

$$\Gamma_k \sim \lambda_k \psi_x^* t_x^* t_x \psi_x$$

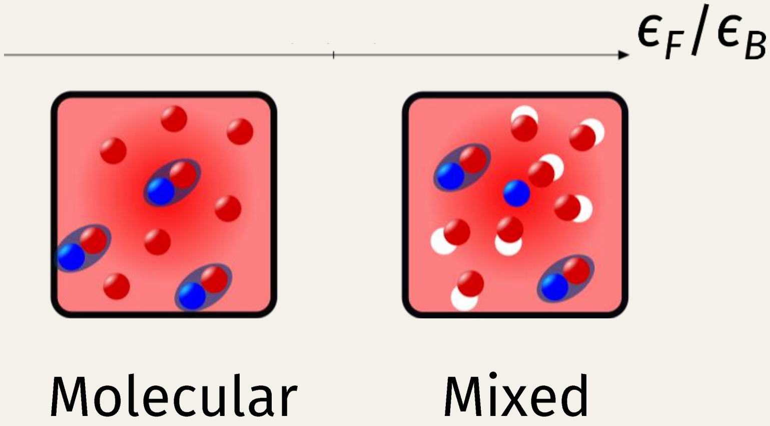
- $\tilde{\lambda}$: 3-body vertex, attractive fermion-molecule interaction.
- Molecule: ϵ_B large → mean-field correct.
- Polaron: suppression of $\tilde{\lambda}$. In-medium effects.
- Molecule energy: → 0 linearly at "transition".



($\tilde{\lambda}$: rescaled by $h_{k=0}$ to get $h \to \infty$ limit)

Finite boson density?

- Limit $0 < n_B \ll n_F$: fermion bath not renormalized

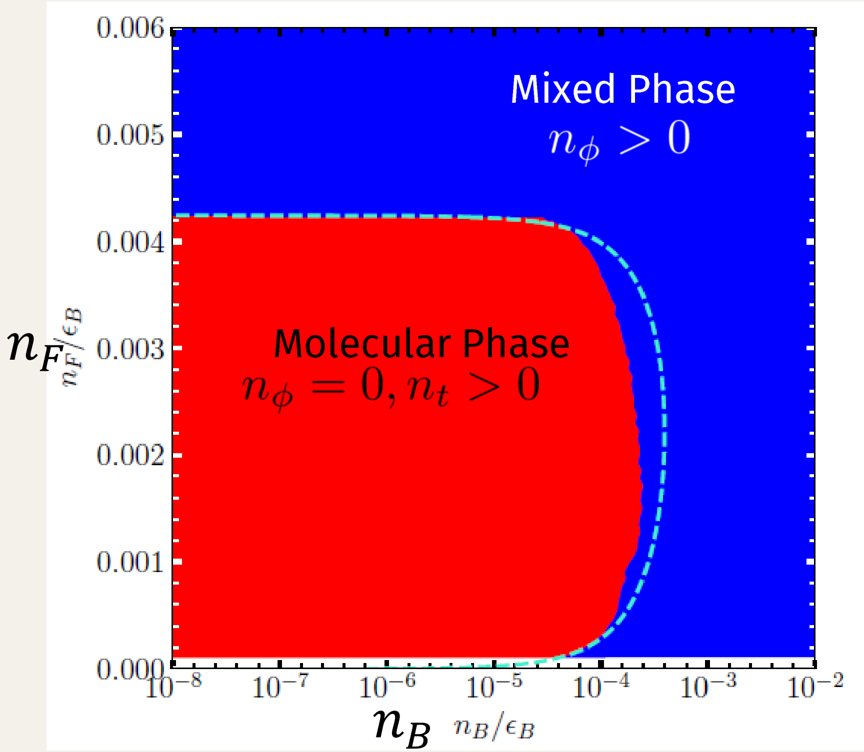


- Molecular phase:
  - All bosons → molecules
    $n_t > 0, n_\phi = 0$

- Mixed phase:
  - Coexistence polaron / molecules
    $n_t > 0, n_\phi > 0$

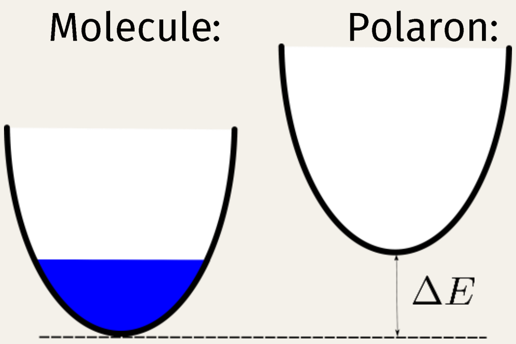
- No pure polaron phase: mixing $h\sqrt{n_\phi}(t^* \phi + \mathrm{h.c.})$ → hybridization

Phase diagram



Simple mean-field picture

$$H^{MF} = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} \psi^\dagger_{\mathbf{k}} \psi_{\mathbf{k}} + \frac{\varepsilon_{\mathbf{k}}}{2} t^\dagger_{\mathbf{k}} t_{\mathbf{k}} + (\varepsilon_{\mathbf{k}} + \Delta E(\epsilon_F/\epsilon_B)) \phi^\dagger_{\mathbf{k}} \phi_{\mathbf{k}}$$



$$\frac{n_B}{\epsilon_B} = \frac{\Delta E(\epsilon_F/\epsilon_B)}{2\pi\epsilon_B}\,, \quad \frac{n_F}{\epsilon_B} = \frac{\epsilon_F}{4\pi\epsilon_B} + \frac{\Delta E(\epsilon_F/\epsilon_B)}{2\pi\epsilon_B}$$

Interactions: destabilise MF towards mixed phase

Spectral functions

Difficulty: analytic continuation $i\omega_n \to \omega + i\,0^+$.

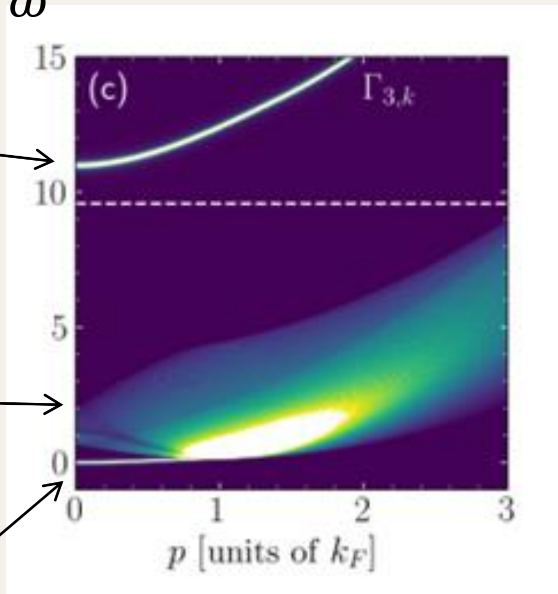
Polaron

ω

Repulsive polaron

Polaron-molecule continuum

Attractive polaron



$\epsilon_B/\epsilon_F = 10$

Solution: continuation of the flow equations.

In practice: solve flow eqns. in 2 steps.

1st step: as before, derivative expansion

2nd step: no projection onto $(p, \omega) = 0$.

Conclusion

- Polaron / mixtures: rich strong coupling physics.
- FRG: valuable tool to explore mixtures, can include bound state / *n*-body correlations.
- Outlook:
  - Bosons: condensate description, effect on superfluidity
  - Fermions: momentum dependence of vertices → Fermi surface topology, bath renormalization at $n_B \sim n_F$...

Collaborators:

Jonas von Milczewski

Richard Schmidt

Questions?

arXiv:2104.14017,
Phys. Rev. A 105, 013317 (2022)

Vertex rescaling

- We assume $h \to \infty$ for equivalence of 1- and 2- channel models.
- Results invole $h \to$ rescaling!
- E.g. 3-body limit:

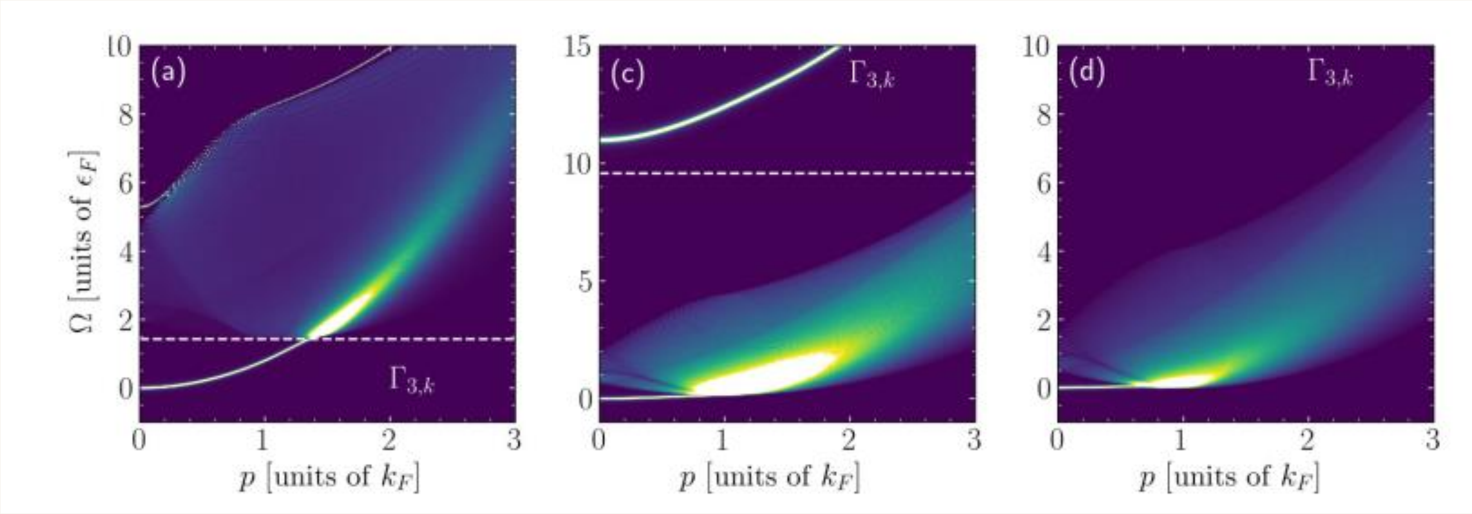
$$m_{t,k} = \frac{h^2}{8\pi} \log\left(1 + \frac{2k^2}{\epsilon_B}\right),$$

$$A_{t,k} = 1 + \frac{h^2}{8\pi}\left(\frac{1}{\epsilon_B + 2k^2} - \frac{1}{\epsilon_B + 2\Lambda^2}\right).$$

For $h \to \infty$, then $\lambda_{k=0} = -h^2/\epsilon_B$

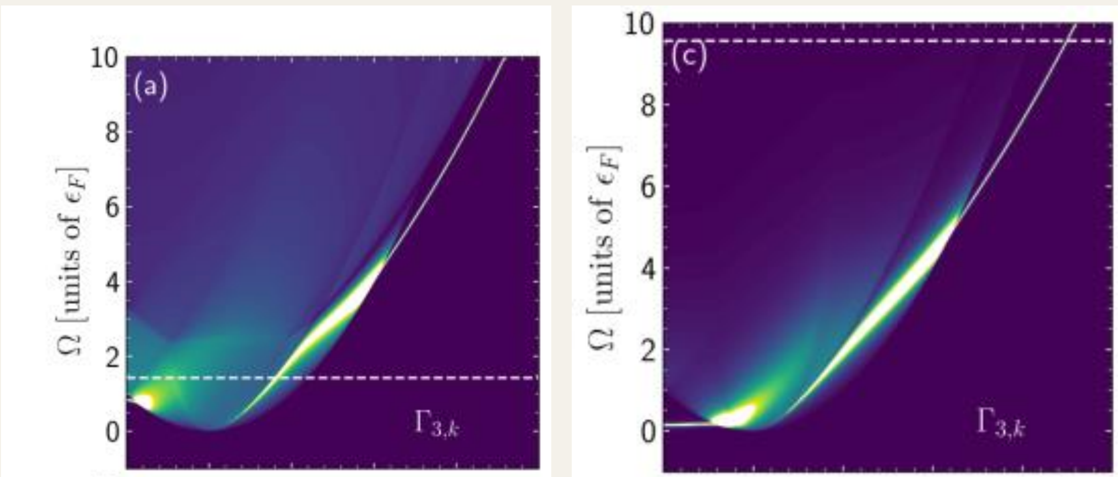
$$\tilde{\lambda} = \frac{\lambda_{k=0}\epsilon_F}{h_{k=0}^2}, \qquad \widetilde{m_t} = m_{t,k=0}/h_{k=0}^2$$

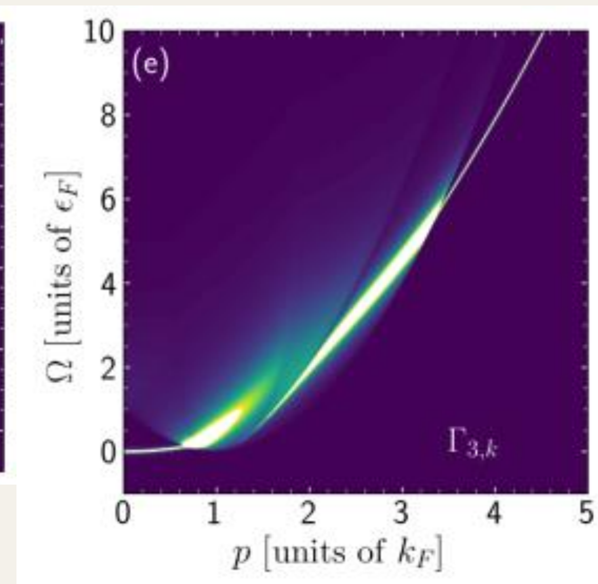
Polaron SF



$$\frac{\epsilon_B}{\epsilon_F} = 1, 10, 20$$

Molecule SF





$$\frac{\epsilon_B}{\epsilon_F} = 1, 10, 20$$