Critical Casimir forces and quantum phase transitions

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Outline of the talk

"Critical Casimir forces": a classical statistical physics problem...

...on which quantum physics allows us to gain insight!

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- What are these critical Casimir forces, and why are they interesting?
- An exact mapping onto a quantum problem.
- Results and conclusion.

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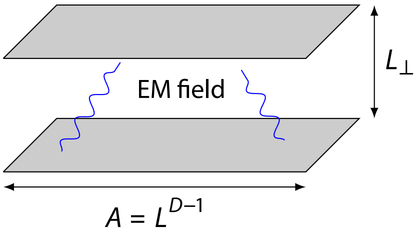
Introduction: the Casimir effect in QED

Consider two parallel metallic plates in vacuum in D dimensions.



Vacuum energy: $E_0 = \frac{\hbar}{2} \sum_{\mathbf{k}} \omega_{\mathbf{k}}$.

Authorized $\mathbf{k}$ depend on boundary conditions $\Rightarrow E_0$ depends on $L_\perp$.

The confinement of the EM field is the source of the Casimir force

$$f = -\frac{1}{A}\frac{\partial E_0}{\partial L_\perp}.$$

[H.B.G. Casimir, Proc. K. Ned. Akad. Wet. 51 (1948) 793.]

The Casimir effect in QED: more insight

To sum up: two ingredients for the Casimir force.

- Vacuum fluctuations: $E_0 = \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}}/2 \neq 0$.
- Boundary conditions (BCs): $\sum_{\mathbf{k}} \to \sum_{\mathbf{k}\,\text{authorized}}$.

Idea: observe the same effect in confined systems near a bulk second order phase transition.

- Correlations over long ranges ($\xi \gtrsim L_\perp$) $\Rightarrow$ boundary effects.
- Thermal fluctuations play the role of quantum fluctuations.

In addition: universality and scaling laws.

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Statistical physics

Functional integrals can describe the long-distance physics of phase transitions.
Eg.: the Ising model

discrete variable s_i ~ spin

$$\mathcal{Z} = \sum_{\{s_i\}} e^{-H[\{s_i\}]}$$

Continuum limit
$\rightarrow$

field $\varphi(\mathbf{x})$ ~ magnetization

$$\mathcal{Z} = \int \mathcal{D}[\varphi]\, e^{-H[\varphi]}$$

Phase transition

$$\begin{cases} \langle \boldsymbol{\varphi} \rangle = 0 & \text{in the symmetric phase,} \\ \langle \boldsymbol{\varphi} \rangle \neq 0 & \text{in the broken symmetry phase (long range order).} \end{cases}$$

Casimir force in classical physics

SFT in D dimensions with a finite length $L_\perp$:

$$H = \int \mathrm{d}^D\mathbf{x}\,\mathcal{H} \quad \to \quad \int \mathrm{d}^{D-1}\mathbf{x}_{\|} \int_0^{L_\perp} \mathrm{d}x_\perp\,\mathcal{H}.$$

The free energy F now depends on $L_\perp$ in addition to $t = (T - T_c)/T_c$.

Free energy $F(t, L_\perp)$ $\Rightarrow$ Casimir force $f(t, L_\perp) = -\partial F/\partial L_\perp$.

In the vicinity of a 2^{nd} order phase transition:

$\xi \sim |t|^{-\nu} \to \infty$, $L_\perp \gg$ microscopic length scales.

Dimensional analysis:

$$\text{excess free energy density} \sim L_\perp^{-D}\mathcal{F}(L_\perp/\xi),$$

$$\text{Casimir force} \sim L_\perp^{-D}\theta(L_\perp/\xi)$$

$\mathcal{F}$, θ: universal scaling functions

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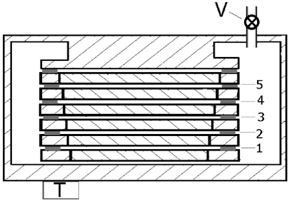
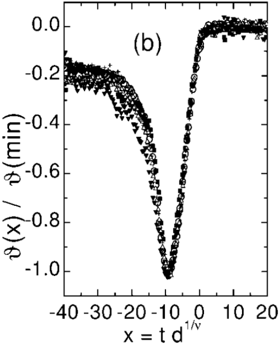
Experiment

Example: liquid ^{4}He films near the λ transition. (3*D XY* universality class).

[R. Garcia and M. H. W. Chan, Phys. Rev. Lett. 83, 1187 (1999).]





Setup: the thickness of adsorbed films is shifted from its expected value ⇒ evidence of the Casimir force.

Quantum phase transitions — examples

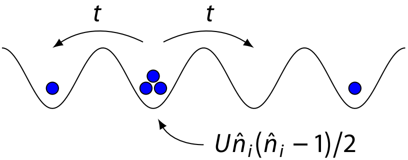
Transverse field Ising model: $\hat{H} = -J_z \sum_{\langle ij \rangle} \hat{s}_i^z \hat{s}_j^z - h \sum_i \hat{s}_i^x$

$(J_z/h)_c$ $\qquad$ J_z/h

Single paramagnetic GS: All spins aligned with **h** field	Degenerate ferromagnetic GSs: All spins parralel to z axis

Interacting lattice bosons (integer filling):

t: hopping
U: on-site interaction energy



$(t/U)_c$ $\qquad$ t/U

Insulating phase (localized bosons)	Superfluid phase (condensate)

Statistical field theory and QPT

Quantum phase transitions can be studied using field theoretical tools.

Second quantization

$\hat{H}, \hat{\psi}(\mathbf{r})^\dagger, \hat{\psi}(\mathbf{r})$ operators

$$\mathcal{Z} = \mathrm{Tr}\, e^{-\beta \hat{H}}$$

Path integral formulation

$\psi(\mathbf{r}, \tau)$ complex fields

$$\mathcal{Z} = \int \mathcal{D}[\psi^*, \psi]\, e^{-S[\psi^*, \psi]}$$

$$\hat{H}[\hat{\psi}^\dagger, \hat{\psi}] \to S[\psi^*, \psi] = \int_0^{\hbar\beta} d\tau \left\{ H[\psi^*, \psi] + \int d^d\mathbf{r}\, \psi^* \partial_\tau \psi \right\}.$$

Periodic BCs: $\psi(\mathbf{r}, \tau + \hbar\beta) = \psi(\mathbf{r}, \tau)$.

- Transforms the d-dimensional quantum problem into a $d + 1$ classical field theory...
- ... at the cost of a new imaginary time dimension $\tau \in [0, \hbar\beta]$.
- A QPT manifests as a phase transition in the classical field theory.

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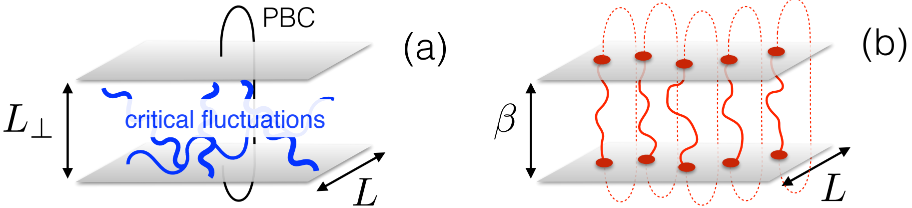
Classical — quantum mapping

Classical $L_\perp < \infty$ theory and quantum $T > 0$ theory are equivalent!

$$H_{\text{cl.}} = \int d^{D-1}\mathbf{x}_{||} \int_0^{L_\perp} dx_\perp \,\mathcal{H}, \qquad H_{\text{q.}} = \int d^d\mathbf{r} \int_0^{\hbar\beta} d\tau\,\mathcal{H}.$$



Scaling functions $\mathcal{F}$, θ now respectively describe the temperature contribution to free energy and to internal energy.

$$F = F_{T=0} + (k_B T)^{d+1}/(\hbar c)^d \mathcal{F}(\Delta/k_B T),$$

$$\epsilon = -\partial[\beta F]/\partial\beta = \epsilon_{T=0} - (k_B T)^{d+1}/(\hbar c)^d \theta(\Delta/k_B T).$$

Δ: zero temperature gap.

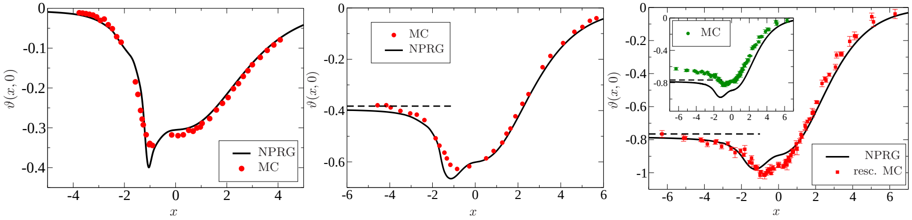
Review and results

Mapping to the quantum system provides new insight into the critical Casimir force with periodic BCs.

- Allows for experiments (ultracold Bose gases, quantum magnets).
- Theoretical methods for quantum systems can be imported!

Our RG-based results for 2d quantum models compare to MC for 3d classical systems!

Left to right: Ising, XY, Heisenberg.



[Rançon et al., *in preparation*].

Questions?