# Hall conductivity and viscosity of a two-dimensional chiral $p + ip$ superconductor

MAX PLANCK INSTITUTE
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# Introduction: chiral superfluids?

Chiral superfluid (SF): Cooper pairing of electrons carrying finite angular momentum.

E.g. spinless 2$d$ fermions w/ $p$-wave interaction: "$p + ip$" SF,

$$\Delta(\mathbf{p}) = -\langle \psi_{\mathbf{p}} \psi_{-\mathbf{p}} \rangle \propto p_x \pm i p_y .$$

→ parity ($P$) and time-reversal ($T$) broken. Angular momentum / particle: $s = \pm 1/2$.

Experimental observations:

- A-phase in $^3$He film under nanoscale confinement.

  [Ikegami et al., Science '13; Levitin et al., Science '13]

- Edge modes of 2$d$ superconductors.

  [Xu et al., PRL '15; Ménard et al., Nature comm. '17]

Phase diagram of a confined $^3$He film (Levitin).

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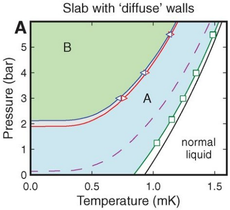



Phase diagram of a confined $^3$He film (Levitin).

# Properties of chiral superfluids

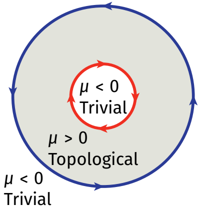
## Topological properties

$$\hat{H} = \int_{\mathbf{q}} \mathbf{\Psi}^\dagger_{\mathbf{q}} \mathcal{H}_{\mathbf{q}} \mathbf{\Psi}_{\mathbf{q}}, \qquad \mathcal{H}_{\mathbf{q}} = \begin{pmatrix} \epsilon_{\mathbf{q}} & \Delta(\mathbf{q})^* \\ \Delta(\mathbf{q}) & \epsilon_{\mathbf{q}} \end{pmatrix} \propto \mathbf{h}(\mathbf{q}) \cdot \boldsymbol{\sigma}.$$

$\mathbf{h}(\mathbf{q})$: mapping from Brillouin zone to the sphere.

$\mathbf{\Psi}^\dagger = (\psi^\dagger, \psi)$,
$\epsilon_{\mathbf{q}} = \mathbf{q}^2/2m - \mu$,
$\boldsymbol{\sigma}$: Pauli matrices.
[Alicea, RPP '12]

- Phase transition between a trivial strongly paired and topological weakly paired phases.
- Chiral Majorana edge states at interface.




Focus of this presentation: transport properties.
*P* and *T* broken → possibility of dissipationless "odd" transport.
[Read and Green, PRB '00]

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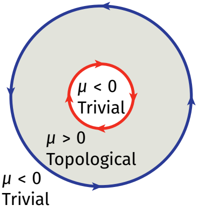



Focus of this presentation: transport properties.

$P$ and $T$ broken → possibility of dissipationless “odd” transport.

[Read and Green, PRB ’00]

# Odd transport coefficients (I): conductivity

(**J**: U(1) current, **E**: electric field)

U(1) charge conservation → conductivity tensor $\sigma^{ij}$

$$J^i = \sigma^{ij} E_j$$

**$\sigma^{ij}$ in a isotropic 2*d* system**

$$\sigma^{ij}(\mathbf{q}) = q^i q^j/\mathbf{q}^2 \sigma_{\mathrm{L}}(\mathbf{q}^2) + (\delta^{ij} - q^i q^j/\mathbf{q}^2)\sigma_{\mathrm{T}}(\mathbf{q}^2) + \epsilon^{ij}\sigma^{\mathrm{H}}(\mathbf{q}^2)$$

$\epsilon^{xy} = 1,\ \epsilon^{ij} = -\epsilon^{ji}$

$\sigma_{\mathrm{L,T}}$: usual longitudinal / transverse conductivities

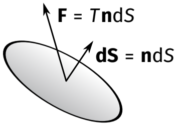
$\sigma^{\mathrm{H}}$: Hall conductivity

- Determines the odd part of conductivity $\sigma^{ij} = -\sigma^{ji}$
- Dissipationless
- Vanishes unless $T$ symmetry is broken

# Odd transport coefficients (II): defining viscosity

Displacement $\partial_i u_j$ and velocity gradients $\partial_i \dot{u}_j$ → internal forces **F**.

Stress tensor $T^{ij}$ fixes the forces.




Internal forces **F** over a surface element **dS**

## Linear response

$$T^{ij} = -\lambda^{ijkl}(\partial_k u + \partial_l u_k)/2 - \eta^{ijkl}(\partial_k \dot{u}_l + \partial_l \dot{u}_k)/2$$

$\lambda^{ijkl}$: elastic modulus, $\eta^{ijkl}$: viscosity

$$\begin{cases} T^{ij} \text{ symmetric} \\ \partial_k \dot{u}_l + \partial_l \dot{u}_k \text{ symmetric} \end{cases} \rightarrow \eta^{ijkl} = \eta^{ijlk} = \eta^{jikl}.$$

## Noether current interpretation

Translation invariance → conserved momentum density, currents $T^{ij}$

$\lambda^{ijkl}$ and $\eta^{ijkl}$: associated transport coefficients

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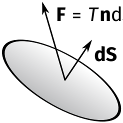


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**Noether current interpretation**

Translation invariance → conserved momentum density, currents $T^{ij}$

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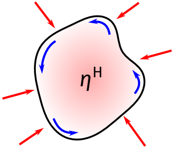
# Odd transport coefficients (III): odd viscosity

Symmetry under exchange of two pairs of indices $(ij)$, $(kl)$: decomposition $\eta = \eta_\mathrm{e} + \eta_\mathrm{o}$.

$\eta_\mathrm{e}^{ijkl} = \eta_\mathrm{e}^{klij}$: even part → bulk, shear viscosities.

$\eta_\mathrm{o}^{ijkl} = -\eta_\mathrm{o}^{klij}$: odd viscosity. [Avron et al., PRL '95]




Velocity field under an external force.

- $\eta_\mathrm{o}$ doesn't exist in $3d$.
- One component in an isotropic system:

$$\eta_\mathrm{o}^{ijkl} = \frac{\eta^\mathrm{H}}{2}(\epsilon_{ik}\delta_{jl} + \epsilon_{jk}\delta_{il} + \epsilon_{il}\delta_{jk} + \epsilon_{jl}\delta_{ik}).$$

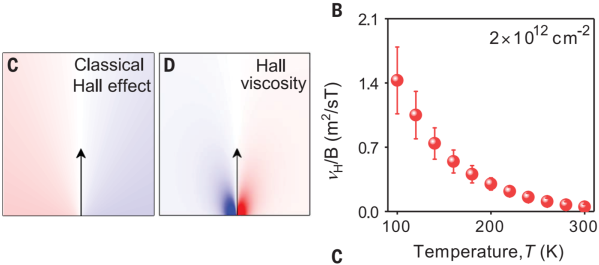
- Dissipationless transport.
- Nonvanishing only if $T$ broken.

$\eta^\mathrm{H}$: "Hall" viscosity

# Hall viscosity in litterature

Recent activity: observation proposals, different systems...

[Lucas and Surowka, PRE '14; Sherafati et al., PRB '16; Delacretaz and Gromov, PRL '17; Scaffidi et al., PRL '17, Tuegel and Hugues, PRB '17]




Observation of (non-quantized) odd viscosity in graphene [Berdyugin et al., Science '19]

(measurement of electric potential near voltage gates)

# Properties in quantum fluids

[Read, PRB '09]: in paired quantum Hall states and chiral SFs,

$$\eta^{\mathrm{H}} = \frac{1}{2}\hbar n_0 s.$$

$n_0$: ground state density
$s \in \mathbb{Q}$: angular momentum / part.

If only one species, U(1) charge transport ≡ momentum density transport
→ Ward identity [Taylor and Randeria, PRA '10; Hoyos and Son, PRL '12; Bradlyn et al., PRB '12]

$$\eta^{\mathrm{H}}(\omega) = \frac{m^2\omega^2}{2}\partial_{q_x}^2 \sigma^{\mathrm{H}}(\omega, \mathbf{q})\Big|_{\mathbf{q}=0}$$

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# Our main result

$\eta^H$ deduced from $\sigma^H$: viscosity-conductivity equivalence?

Above relation not completely general.

- At finite **q**, two independent components $\eta_o^{(1,2)}(\omega, \mathbf{q}^2)$ for isotropic systems

$$\eta_o = \eta_o^{(1)}\sigma^{xz} + \eta_o^{(2)}[(q_x^2 - q_y^2)\sigma^{0x} - 2q_xq_y\sigma^{0z}]$$

[Golan et al., PRB '19]

$$(\sigma^{ab})^{ijkl} = (\sigma^a)^{ij}(\sigma^b)^{kl} - (\sigma^b)^{ij}(\sigma^a)^{kl}$$

- $\eta^{(1)} = -\eta^H$: identified with the "usual" Hall viscosity.
- $\eta^{(2)}$: usually subleading...
- ...unless it is singular! With long-range forces, $\eta^{(2)}(\omega \to 0, \mathbf{q} \to 0) \sim 1/\mathbf{q}^2$

Long-range interactions like 2*d* Coulomb can break down the identity
Viscosity tensor contains more information than Hall conductivity!

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Long-range interactions like $2d$ Coulomb can break down the identity
Viscosity tensor contains more information than Hall conductivity!

# Plan of the presentation

From here on:

- Field theory setup for the calculation.
- Conductivity and viscosity via diagrams.
- Effect of Coulomb.

# Field theory definition of the transport coefficients

Starting point: action S

- U(1) current → electromagnetic field: $J^i \sim \frac{\delta S}{\delta A_\mu}$.
- Momentum current → non-flat metric: $T^{ij} \sim \frac{\delta S}{\delta g_{ij}}$.

($g_{ij}$ ~ displacement gradient)

**Linear response theory**

$$\delta\langle J^i\rangle \sim \sigma^{ij} \overbrace{[\partial_t A_j - \partial_j A_t]}^{\text{electrical field}} \longrightarrow \sigma^{ij} \sim \omega\langle[J^i, J^j]\rangle$$

$$\delta\langle T^{ij}(x)\rangle \sim -\frac{1}{2}\eta^{ijkl} \underbrace{\partial_t g_{kl}}_{\simeq \text{ velocity gradient}} \longrightarrow \eta^{ijlk} \sim \omega\langle[T^{ij}, T^{kl}]\rangle$$

# Induced action

**Induced action**

$$e^{i\mathcal{W}[A,g]} = \int \mathcal{D}\underbrace{[\cdots]}_{\text{fields}}\, e^{iS[\cdots;A,g]}$$

$\mathcal{W}[A,g]$ obtained by integrating out the fluctuating fields.

- $\mathcal{W}$: generating functional of connected correlation functions
  (analog to free energy in stat. mech.!)
- Transport coefficients $(\omega^+ = \omega + i0^+)$

$$\frac{\delta^2\mathcal{W}}{\delta A_i \delta A_j}(\omega,\mathbf{q}) = i\omega^+ \sigma^{ij}(\omega,\mathbf{q}),$$

$$\frac{\delta^2\mathcal{W}}{\delta g_{ij} \delta g_{kl}}(\omega,\mathbf{q}) = \frac{i\omega^+}{4}\eta^{ijkl}(\omega,\mathbf{q}) - \frac{1}{4}\lambda^{ijkl}(\omega,\mathbf{q}) + \frac{1}{4}\langle T^{ij}\rangle\delta^{kl}$$

# Ward identity

Only one species → transport of momentum ≃ transport of U(1) charge.

Formally: $\mathcal{W}$ inherits microscopic symmetries of the original action

- U(1) gauge transforms of parameter $\alpha(t, \mathbf{x})$
- change of coordinates $x^i \to x^i + \xi^i(t, \mathbf{x})$

$$\mathcal{W}[A + \delta A, g + \delta g] = \mathcal{W}[A, g]$$

$$\delta A_t = -\partial_t \alpha - \xi^k \partial_k A_t - A_k \partial_\tau \xi^k,$$
$$\delta A_i = -\partial_i \alpha - \xi^k \partial_k A_i - A_k \partial_i \xi^k + m g_{ik} \partial_t \xi^k,$$
$$\delta g_{ij} = -\xi^k \partial_k g_{ij} - g_{ik} \partial_j \xi^k - g_{kj} \partial_i \xi^k.$$

$m g_{ik} \partial_t \xi^k$: preserve invariance for *time-dependent* transforms

[Son and Wingate, Ann Phys ’06]

Functional derivatives → identities between *n*-point correlation functions

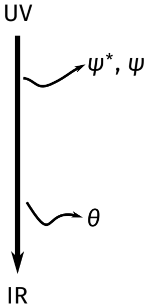
$$m^2 (\omega^+)^2 \sigma_{\mathrm{H}}(\omega, \mathbf{q}^2) = -\mathbf{q}^2 [\eta_o^{(1)}(\omega, \mathbf{q}^2) - \mathbf{q}^2 \eta_o^{(2)}(\omega, \mathbf{q}^2)]$$

[Geracie et al., PRD ’15]

$$\eta_o = \eta_o^{(1)} \sigma^{xz} + \eta_o^{(2)} [(q_x^2 - q_y^2)\sigma^{0x} - 2 q_x q_y \sigma^{0z}]$$

# Microscopic versus effective action

What starting point for the actual calculation?




- Microscopic theory
  Start from fermionic action, contains “everything”

- Effective theory (Son, Hoyos, Moroz)
  Low-energy physics from symmetry breaking
  Simpler, but can be difficult to change setup

Both approaches are complementary; we present here the microscopic calculation.

# BCS action with no sources (I)

**Starting point**

$$S[\psi^*,\psi] = \underbrace{\int_x \psi^*\left[\partial_\tau + \frac{\mathbf{p}^2}{2m} - \mu\right]\psi}_{\text{free fermions}} - \underbrace{\lambda(\psi^*\mathbf{p}\psi^*)\cdot(\psi\mathbf{p}\psi)}_{p\text{-wave interaction } S^{\text{int}}}$$

Interaction decoupled via a pairing field:

$$\bar{\mathbf{\Delta}} = (\bar{\Delta}_x, \bar{\Delta}_y), \qquad \bar{\mathbf{\Delta}} \sim \psi\mathbf{p}\psi$$

$$S^{\text{int}}[\psi^*,\psi] \to S^{\text{int}}[\psi^*,\psi,\bar{\mathbf{\Delta}}^*,\bar{\mathbf{\Delta}}] = \int_x -\frac{1}{2}(\bar{\mathbf{\Delta}}^i)^* \cdot (\psi_x\mathbf{p}\psi_x) - \frac{1}{2}\bar{\mathbf{\Delta}}\cdot(\psi_x^*\mathbf{p}\psi_x^*) + \frac{1}{4\lambda}|\bar{\mathbf{\Delta}}|^2$$

# BCS action with no sources (II)

Analog to *s*-wave superconductivity:

**Grand potential**

$$\Xi[\bar{\mathbf{\Delta}}^*, \bar{\mathbf{\Delta}}] \equiv \int \mathcal{D}[\psi^*, \psi] \exp(-S[\psi^*, \psi, \bar{\mathbf{\Delta}}^*, \bar{\mathbf{\Delta}}])$$

$$\text{Mean-field: } \frac{\Xi[\bar{\mathbf{\Delta}}^*, \bar{\mathbf{\Delta}}]}{\delta \bar{\mathbf{\Delta}}^{(*)}} = 0.$$

$$\Downarrow$$

Saddle-point configuration: $\bar{\mathbf{\Delta}} \cdot \mathbf{p} = \Delta_0 e^{i\theta}(p_x \pm i p_y)$

- $\Delta_0 = 0 \rightarrow$ normal phase, $\Delta_0 \neq 0 \rightarrow$ SF
- U(1) and SO(2) broken, but invariance under diagonal U(1) combination

# Coupling to sources

**U(1) gauge field $A_\mu$**

$$\begin{cases} \partial_\tau \to \partial_\tau - iA_0, \\ \mathbf{p} \to \mathbf{p} - \mathbf{A}. \end{cases}$$

**metric $g_{ij}$**

volume element $\sqrt{g}$

$$\mathbf{a} \cdot \mathbf{b} \to g_{ij} a^i b^j$$

$$\partial_i \to \nabla_i$$

Gaussian theory: discard fluctuations of the pairing field...

...while preserving the symmetries!

- $\bar{\boldsymbol{\Delta}} = e^{i\theta}\boldsymbol{\Delta} \to 4$ excitation modes: $\begin{cases} \text{3 gapped modes} \to \text{discarded.} \\ \text{phase } \theta \to \text{crucial for U(1) gauge invariance.} \end{cases}$

  $\theta$ gauged away via transform $\psi \to e^{i\theta/2}\psi$: $\boxed{A_\mu \to \mathcal{A}_\mu = A_\mu - \nabla_\mu \theta / 2e}$

- In non-flat space, mean-field given by $\boxed{\Delta^i = e^{ia}\Delta_a}$ $\Delta_a = \Delta_0(1, \pm i)_a$

  $g^{ij} = e^{ia}\delta_{ab}e^{ib}$, $e^{ia}$ vielbein ($\simeq$ local base)

# Calculation setup

## Starting point in imaginary time

$$S[\psi, \psi^*; \mathcal{A} = A - \nabla\theta, g] =$$

$$\int_x \sqrt{g}\left\{\psi^*\left[\partial_\tau - ie\mathcal{A}_0 + \frac{g^{ij}(p_i - e\mathcal{A}_i)(p_j - e\mathcal{A}_j)}{2m} - \mu\right]\psi - \frac{1}{2}\Delta_a^* e^{ia}(\psi p_i \psi) - \frac{1}{2}\Delta_a e^{ia}(\psi^* p_i \psi^*)\right\}$$

$$S[\mathbf{\Psi}, \mathbf{\Psi}^\dagger; \mathcal{A}, h] = -\frac{1}{2}\int_{x,x'} \mathbf{\Psi}^\dagger(\mathcal{G}_0^{-1} - \Gamma)\mathbf{\Psi}$$

$\mathbf{\Psi} = (\psi^*, \psi)$ Nambu spinor

- $\mathcal{G}_0(q) = \begin{pmatrix} G(q) & F(q) \\ F(q)^* & -G(-q) \end{pmatrix}$ bare propagator in absence of sources.
- $\Gamma$: contribution from the sources → vertex.

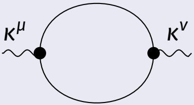
Loop expansion: $S[\mathcal{A}, g] = -\operatorname{Tr}\log(-\mathcal{G}_0^{-1} + \Gamma) = -\operatorname{Tr}\log\mathcal{G}_0 + \operatorname{Tr}\mathcal{G}_0\Gamma + \frac{1}{2}\operatorname{Tr}\mathcal{G}_0\Gamma\mathcal{G}_0\Gamma + \dots$

# Conductivity

$$S[\mathcal{A} = A - \nabla\theta] = \frac{1}{2}\sum_q \mathcal{A}_{-q,\mu} Q^{\mu\nu}(q) \mathcal{A}_{q,\nu}.$$

**Polarization bubble**

$Q^{\mu\nu} \sim \operatorname{Tr} \mathcal{G}\Gamma\mathcal{G}\Gamma \sim$

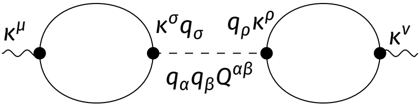



$\Gamma \sim \kappa^\mu \mathcal{A}_\mu$

$$\int \mathcal{D}[\theta] \to \boxed{\mathcal{W}[A] = \frac{1}{2}\sum_q A_{-q,\mu} \underbrace{\left(Q^{\mu\nu}(q) - \frac{Q^{\mu\rho}(q) q_\rho q_\sigma Q^{\sigma\nu}(q)}{q_\alpha q_\beta Q^{\alpha\beta}(q)}\right)}_{K^{\mu\nu}(q)} A_{q,\nu}.}$$

[Lutchyn et al., PRB' 08]

$K^{\mu\nu}(q) \sim$




**$f$-sum rule**

$q_\mu K^{\mu\nu} = 0 \to$ gauge invariance

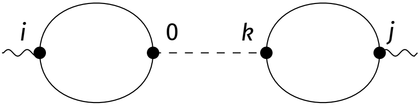
# Conductivity — result

**Small $q$**

$$\sigma_{\mathrm{H}}(\omega, \mathbf{q}) = \hbar \frac{s n_0}{2m^2} \frac{-\mathbf{q}^2}{\omega^2 - c_s^2 \mathbf{q}^2}$$

$s = \pm 1/2$, $n_0$ density, $c_s$ sound velocity

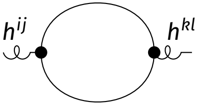
- In agreement with effective field theory. [Hoyos et al., PRB ’13]
- phase mode crucial for $\sigma_{\mathrm{H}} \neq 0$.
- Hall conductivity appears through odd part of density-current correlator $Q^{i0}(q)$.



# Viscosity

Similar setting, $\mathcal{G}^{-1} = \mathcal{G}_0^{-1} - \Gamma$, $\Gamma$: gravity-fermion vertex.

- Phase mode plays no role.
- Only contribution: bubble.




Result:

**Small $q$**

$$\eta_{\rm H}(\omega, \mathbf{q}) = -\hbar \frac{s n_0}{2}$$

- Basic Ward identity satisfied.
- Finite $\mathbf{q}$: Ward needs $\eta_o^{(2)}$.
- $\eta_o^{(2)} \rightarrow$ EFT (for the moment).

# Adding Coulomb interaction (I)

Coulomb interaction ~ $e^2 \int_{x,x'} (\psi^*\psi)_x V_{x-x'} (\psi^*\psi)_{x'}$ decoupled via field $\chi$.

**Coulomb action**

$$\frac{1}{2}\int \boldsymbol{\nabla}\chi \cdot \boldsymbol{\nabla}\chi - ie\int \chi\psi^*\psi$$

Coupling to fermions: $\begin{cases} A_0 \to \tilde{A}_0 = A_0 + \chi \\ A_t \to \tilde{A}_i = A_i \end{cases}$

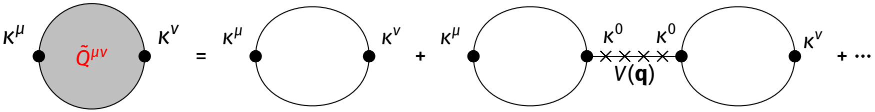
$$S[\tilde{\mathcal{A}} = \tilde{A} - \nabla\theta] = \frac{1}{2}\sum_q \tilde{\mathcal{A}}_{-q,\mu} Q^{\mu\nu}(q) \tilde{\mathcal{A}}_{q,\nu}$$

$$\Downarrow \int \mathcal{D}[\chi]$$

$$S[\mathcal{A} = A - \nabla\theta] = \frac{1}{2}\sum_q \mathcal{A}_{-q,\mu} \tilde{Q}^{\mu\nu}(q) \mathcal{A}_{q,\nu}$$

$$\tilde{Q}^{\mu\nu} = Q^{\mu\nu} + \frac{V_{\mathbf{q}}}{1 - V_{\mathbf{q}} Q^{00}} Q^{\mu 0} Q^{0\nu}$$

RPA resummation of Coulomb



# Adding Coulomb interaction (II)

Conductivity: adding Coulomb opens up a plasmon gap $\omega_\mathrm{p} = (n_0 e^2/m)^{1/2}$.

**Small $q$**

$$\sigma_\mathrm{H}(\omega, \mathbf{q}) = \hbar \frac{s n_0}{2m^2} \frac{-\mathbf{q}^2}{\omega^2 + \omega_\mathrm{p}^2 - c_\mathrm{s}^2 \mathbf{q}^2}$$

$$\partial_{q_x}^2 \sigma_\mathrm{H} \sim \begin{cases} 1/(\omega^+)^2, & \text{no Coulomb} \\ \text{const.} & \text{Coulomb} \end{cases}$$

Viscosity: no coupling to Coulomb through the metric-fermion vertex.

$\eta^\mathrm{H}$ unchanged $\rightarrow \frac{m^2\omega^2}{2} \partial_{q_x}^2 \sigma^\mathrm{H} \Big|_{\mathbf{q}=0} = 0 \neq \eta^\mathrm{H}$.




Subleading at RPA level.

Only way to reconciliate Ward identity: divergence of $\eta_\mathrm{o}^{(2)}$.

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# Adding Coulomb interaction (II)

Conductivity: adding Coulomb opens up a plasmon gap $\omega_p = (n_0 e^2/m)^{1/2}$.

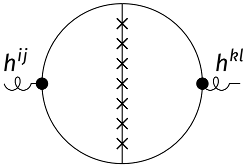
**Small $q$**

$$\sigma_H(\omega, \mathbf{q}) = \hbar \frac{s n_0}{2m^2} \frac{-\mathbf{q}^2}{\omega^2 + \omega_p^2 - c_s^2 \mathbf{q}^2}$$

$$\partial_{q_x}^2 \sigma_H \sim \begin{cases} 1/(\omega^+)^2, & \text{no Coulomb} \\ \text{const.} & \text{Coulomb} \end{cases}$$

Viscosity: no coupling to Coulomb through the metric-fermion vertex.

$\eta^H$ unchanged $\rightarrow \frac{m^2\omega^2}{2} \partial_{q_x}^2 \sigma^H \Big|_{\mathbf{q}=0} = 0 \neq \eta^H$.

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# Summary and overlook

2$d$ Coulomb → $\eta_o^{(2)} \sim 1/\mathbf{q}^2$ divergent.

Ward identity broken; Hall conductivity—viscosity inequivalence

- Shown with microscopic and effective action.
- Article in preparation!
- Remaining questions: 3$d$ Coulomb, static limit, retardation effect.
- 2$d$ Coulomb realized in Josephson junction arrays. [Mooij et al., PRL '90]
- Application: theories for FQHE $\nu = 1/2$ state? [Son, PRX '15]

Thanks for your attention!

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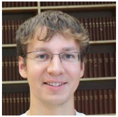
Collaborators



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